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Geomistory: or An Incompatible Synthesis

Nothing feels more odd than when you realize that you have proved yourself to be completely wrong. The original assumption that this paper was based on relied on the simple thought that geometry and history were essentially similar in many ways. Just as other people find similarities in mathematics and science and use that as evidence as to why they understand them both, I attempted to do so with geometry and history. In this case evidence ran contrary to intuition. Certain elements fell into line: the forming of the hypotheses, the logical way conclusions are approached, and the way they both have greater applications to life. While geometry seems similar on the surface to history, and even as they can parallel each other in certain respects, the type of logical progression, the reliability of evidence and communication, and the definitive nature of the conclusions are all quite dissimilar.

Simplicity, elegance, agreement – all these concepts are highly desired by both the geometer and the historian. Those concepts point to correct means and the end is success. Success can only be displayed through a proof. The need for a proof, as an idea, transcends subject matter and hence can be used as a basis of comparison between history and geometry. The similarity between geometry and history begins with the logical structure of hypotheses. The basic things with which hypotheses are based from are

axioms and primary sources respectively. Both geometry and history build on past examples to improve or disprove what others have already accomplished. When a hypothesis is created it requires a certain amount of creativity, and the only things that the geometer/historian can use are the axioms/primary sources, or past results proved from these such as postulates/secondary sources. Some difference exists in regards to history, as secondary sources may not be totally reliable, but that subject will be addressed at a later point on this paper. It is in how the proofs conclude that the main differences show themselves, as the validity of conclusions is the most immediate judge of success.

Success is a definite thing in math. There cannot be an incomplete and yet accepted proof. The steps must all be justified for success to occur. One infamous example is Proclus' attempt to deduce Euclid V from the axioms of neutral geometry. If every step but one is justified, then that step is implied to be equivalent to Hilbert's parallel postulate and hence also to Euclid V, making the proof redundant. Therefore, for a proof to be complete and successful, all the steps must be justified. However, success is something that is not so restrictive as the mere need to have a complete and effective proof based on what the geometer was originally trying to prove. The common example of how, on the way to proving one thing, other useful results are found, is key to understanding how geometry can have many levels of success. Lemmas, and even postulates have been formed this way, often while geometers were in search of the elusive proof of Euclid V. Failure can then only occur when a hypothesis is wrong and nothing else is gained along the way. Each part of geometry builds off the rest, and whatever you discover only helps future geometers have a diverse understanding of the subject. This is part of what makes geometry so beautiful in structure: it continues to

move forward. As more is discovered and thought about it merely gets added to the whole making it a constructive act.

History has potential for being both constructive and destructive to past hypotheses. Any number of hypotheses could be made based upon the evidence given, as long as the facts, or historical conclusions, are not different. Wildly varying interpretations could be valid within the context that the individual historian gives. One reason for this is that not every person has access to all the facts all the time. New information is always being sought and discovered, and old documents are compared and questioned so that their content can be considered to be factual with a high degree of confidence. This is one reason history is a subject that has a potential for great change and controversy. If a new document surfaces then suddenly a whole concept of what happened could shift. In the example of World War I, if it were not known that Germany was allied with Austria then the hypotheses of why they entered into the war would be different. A person could make an argument about how it was the climate at the time to feel kinship towards the Austrians, or maybe that it was economically more rational. If, in the midst of that, the knowledge of the alliance sprang up, then stronger hypotheses would develop that could either contradict or build off earlier ones. The strongest hypothesis depends what the individual felt was the strongest and best backed up theory.

The levels of success and failure in history are layered and not definite. So, success is much more difficult to achieve and hold on to than the successes of geometry. Success only happens in history when the majority accepts a given theory; a consensus is difficult to arrive at and takes some time and debate. Failure is much more likely in this context because there could be equal and mixed opinions about a theory's validity, or it

could not be justified by enough of the right kind of evidence. However, even this can be considered somewhere in-between a success and a failure, because some opinions rise and fall over time as more analysis happens. A commonly held belief could be "proved", "disproved", and again reaffirmed within a short period of time, such as the Sally Hemming/Thomas Jefferson debate that took place a few years ago. Documents and the evidence gained from them alone can sometimes be sketchy. The definition of success itself also depends on the individual, and the one above is one I created merely so that it could be more easily understood within the context of this paper. It is entirely possible that success could be redefined to be more static like geometric success, but the interpretive level of it does not change.

Proof is tightly interlocked with communication in that there must be things about the basic facts that everyone can agree on within a given subject. Geometry solved this problem for within itself by making undefined terms and axioms. The undefined terms serve as a sort of jumping off point. They are amorphous building blocks for a definite structure, and cannot be defined because they are just words that make it more convenient to describe relationships between concepts. The first three undefined terms, point, line, and incidence, and the accompanying axioms can be modeled in so many ways that what they are is not important so much as that the conclusions based from their relationships are important. If a person can use terms or symbols that don't actually mean anything in particular to describe a relationship whose parameters are exact in a way that other people cannot refute and based upon knowledge of past relationships, then successful communication has taken place in geometry. The axioms, work within this system as the initial rules of conduct between the undefined terms. They are facts that are so basic that

they seem to need no direct proof in of themselves. Axioms are intuitive; undefined terms are also intuitive in that we have a sense of what they do even if we don't know what they are. Hence, communication in geometry is based on many parts we accept leading to concepts that are less intuitive but nevertheless deductively true.

Communication of proof in history is almost opposite to that in geometry. The end is already known, as if someone already told the end of the novel and now we need to figure out how it happened. Take World War I and how it began as an example. The war has happened already and in fact ended a long time ago so there are few people left who remember it first hand. All that leaves as primary sources are letters, reports, and various other documents. Certain facts are known – Franz Ferdinand was shot, and then war soon erupted – but where are the details? In some ways this working backwards is like trying to find the axioms from the theorems. What are the most basic and reasonable things that could have led to the conclusion? Intuition backwards; as a concept it is odd to phrase, but essentially that is what reconstructing an event becomes. Communication only results in success when there are enough effects to warrant the conclusion, or cause, and the synthesis of it all is what historical texts are about. The main problem of communication is that although the majority of the time the facts of the evidence are easy enough to agree on, it is so open to interpretation and complex that the "how" and the "why" both get confused.

Few points are more confusing than the "why" of history. Meaning is difficult to pin down in interpretive concepts such as history. This seems almost incomprehensible since there must be a finite number of reasons. This is true, but the fine print does not say how many there are. There could be a few, a hundred, a thousand and any number of

them could have had a significant impact. The reasons appear to work in chains and web structures simultaneously, each factor playing off previous ones as well as current ones. Take this, for example: if Franz Ferdinand had not been vain he would not have had himself sewn into his suit, which perhaps would have allowed him to get medical attention that would have saved his life. If he had not died then maybe Austria would not have been upset enough to declare war and feel the need to call in the favor with its alliance to Germany and thereby embroil Europe in war. This happened at a time when the various powers of Europe were all competing for industrial and martial dominance in numbers, and mobilization was at the tip of everyone's tongue. If there weren't the same amount of tension, then maybe the war would not have happened. There were a lot of points and qualifiers in those sentences, and I only scratched the surface because the political background that led up to the tension also developed over years, as did the increased industrialization of Europe. Ultimately, it is up to the historian what should be considered the primary reasons out of the many that exists. Even knowing this the quality of the conclusion in a history paper relies on the quality and quantity of the evidence, and the bias of the historian. Certain reasons are discarded as unjustifiable, or ridiculous, mostly based on how the person interprets the sources. Context, purpose, and comparative facts are used to evaluate the reliability of sources, but even with that bias is still a major consideration. It is bias and point of view that colors history with controversy. If all the facts were known and recorded without any bias at all, there would still be argument about what it meant and why it happened that way because everyone does not think the same way.

Bias is not only the problem of historians. Geometry experiences bias in a number of ways. Ideology that certain things are possible and others not set back the development of hyperbolic geometry. The bias that Euclidean geometry was the "real" geometry was the particular reason. Girolamo Saccheri, a noted mathematician in the 1600s, when he could not prove Euclid V using quadrilaterals then exclaimed it "repugnant to the nature of the straight line"[1] that Euclid V was not true and left it alone for others to consider. Beyond that personal bias about the interpretations of geometry, meaning usually takes a very structured form. Meaning by its connotation implies something with infinite possibilities. At first meaning and structure can feel contradictory, but they do not have to be. Multiple ways to interpret an axiom exist, and it is from this that the hypotheses form. The literal use of an incidence axiom could only supply a few points and a few lines, but the interpreted concept gives us the tools to make postulates, theorems, and from those we can construct a world of neutral geometry not limited to our own finite conception of the world. Merely knowing that for every two distinct points there exists a unique line incident on them (IA-1), does not give the full picture when taken literally. There are two points and a line that way. From the two points we get an infinite number of lines, all types as directions, as long as we can find two points. The breadth of it is startling. The form of having a concept X leading to related concepts W, Z, and Y is the spirit behind this thought of the meaning of geometry.

The essential question that unites these diverging subjects lies in whether meaning is truth. Truth as a concept is something that again can take on wildly differing definitions based on the person you ask. Even as this is correct about truth, there seems

[1] P.155, Greenberg, Marvin Jay, *Euclidean and Non-Euclidean Geometries: History and Development*, W.H. Freeman and Company, New York, 1999

to be some drive in the human as a thinking creature to try to discover if there is a truth that carries through every single person's definition. Truth should not be divided, for as truth it implies a singular. History and geometry both search for an ultimate truth even as there may not be one. Geometry strictly regulates its truth, to a certain extent, by saying that a well-formed statement can either be true or false but not both at the same time. As it is structured in this way, statements that are proven to be true are directly deduced from the axioms and hence also become true. In this sense, neutral geometry is true.

Even in geometry, however, there is enough of an evaluative element to put truth in question. The different types of geometry formalism, intuitionism, and logicism disagree about what should be allowed as true. Formalism, as the most generally and widely practiced, is the traditional axiomatic method. I am taking formalism as the basis for my understanding as truth in mathematics, whereas much of what I have said preceding this might have been false if I were, say, an intuitionist. It seems the most logical to me, and the majority of mathematicians, to use formalism so it is the "truth" of geometry. Beyond different concepts of neutral geometry, the other concept of truth in geometry that comes into question involves the parallel postulate. Historically, geometers wanted Euclid V to be true. It was on the tip of their proverbial tongue, or so they thought, and two thousand plus years went towards trying to make that the truth of what geometry was. Eventually, a new concept of truth for our world was conceived and developed in hyperbolic geometry. Now that we can say with certainty that both Euclidean and hyperbolic geometry are either both consistent or both flawed, and only one can exist at a time, but our tools are not good enough to determine if we live in a Euclidean or hyperbolic world. This is where the ultimate truth leads for geometry: our

concept of our world and how it works. To a certain extent, both geometers and historians make their own worlds, each as real and viable as ones created before, with the difference being that geometers can be surer of their foundations.

History is not so lucky to have something so easily pinned down in regards to the nature of truth. The sources, which are hopefully honest in that what they record is factual, are what truth is deduced from. A good historian tries to take into account that any one source may not be accurate and so use as many sources as possible, so this aspect is usually minimized. Just as meaning and proof cannot be definite within history, truth shall also reflect that imprecision. A well-formed statement in history is also given in a form where it is either true or false, but, unlike geometry, it is possible to justify a statement, call it true, and have it be false. This is impossible to do in an axiomatic method, which is what geometry is commonly defined in. So, truth can be sought fervently in history but never defined as an absolute. It is peculiar to think of it as such a shifting thing because the way history is taught to people is through teachers who are only giving the most common and acceptable interpretation of the time. Even if the guesses are good, even logical, there is no way to definitively prove most historical hypotheses unless it can be proved through some other sort of field, such as science.

Yet, the purpose of finding truth in history is to understand why we do the things we do, and how we can prevent negative things from happening again in the future. Human motivation may not be something we can give with direct certainty, but there is a sort of definite truth in knowing how people will react based on past actions. In this respect history serves its purpose in expressing truth: people relive history, and we can learn from it each time. In World War I, there were many alliances that forced the

countries to go to war. These pacts, when looked at again, are now recognized as a bad idea in many circumstances. Another thing learned from the war is the cost to power, because it was due to the countries of Europe fighting amongst themselves that the United States rose to power as an arms dealer, hence Europe lost supremacy. Another greater and more tragic cost was the lives of a generation of men, and the development of new and more dangerous technology. These aspects had all been present in previous wars, but the extreme to which they were taken provides a good look at the rise of modern warfare and its dangers. So history works as a tool to understand humanity as a whole, even if we cannot be certain about one event in particular. It is the accumulation of events that leads to the conclusion.

Geometry initially looks similar to history in certain ways. There is the presence of definite logic, where a person must supply a hypothesis and conclusion, and use evidence to prove it… but that is where the similarities end. Geometry moves forward, it builds continually so that with each passing year it becomes that much more rich, that much more certain. History, as time progresses often backtracks on itself with conflicting, but supported, theories. The way in which a person comes to a conclusion in geometry builds up a conclusion based on past knowledge and can lead to any number of other discoveries. While there are discoveries in history, they are all on the causal side because the conclusion must lead to a known fact. Both history and geometry are a search for an ultimate truth, the former about the nature of human beings and the latter about the nature of the world around us. In the common search for truth they can show us things about our world that previously seemed incomprehensible, even if those truths are so different in base concept.

Bibliography

1. Buxton, Laurie, *Mathematics for Everyone*, Schocken Books, New York, New York, 1985

2. Falls, Cyril, *The Great War*, Capricorn Books, New York, New York, 1959

3. Koch, H. W., *The Origins of the First World War: Great Power Rivalry and German War Aims*, Taplinger Publishing Company, New York, New York, 1972

4. Montague, Harriet & Montgomery, Mabel, *The Significence of Mathematics*, Charles E. Merrill Books Inc., Columbus, Ohio, 1963

5. Repington, Colonel Charles, *The First World War 1914-1918, Vol. 1*, Houghton Mifflin Company, Boston and New York, 1920